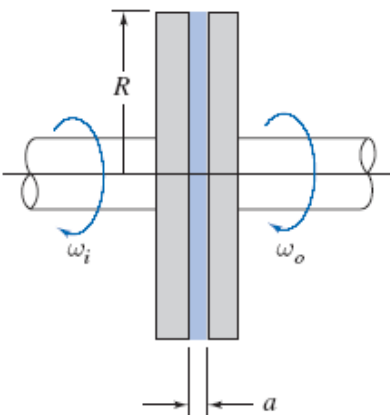


Problem 2.71

[Difficulty: 5]

2.71 A viscous clutch is to be made from a pair of closely spaced parallel disks enclosing a thin layer of viscous liquid. Develop algebraic expressions for the torque and the power transmitted by the disk pair, in terms of liquid viscosity, μ , disk radius, R , disk spacing, a , and the angular speeds: ω_i of the input disk and ω_o of the output disk. Also develop expressions for the slip ratio, $s = \Delta\omega/\omega_i$, in terms of ω_i and the torque transmitted. Determine the efficiency, η , in terms of the slip ratio.



Solution: Apply Newton's law of viscosity

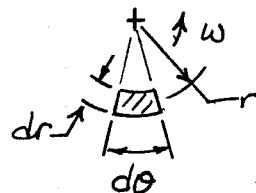
Basic equations: $\tau = \mu \frac{du}{dy}$ $dF = \tau dA$ $dT = r dF$

Assumptions: (1) Newtonian liquid
(2) Narrow gap so velocity profile is linear

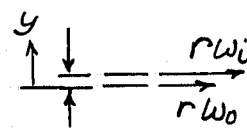
Consider a segment of plates:

$$\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y} = \mu \frac{r(\omega_i - \omega_o)}{a}$$

$$dA = r dr d\theta$$



End View



Bottom View

$$dF = \tau dA = \frac{\mu r \Delta\omega}{a} r dr d\theta = \frac{\mu \Delta\omega}{a} r^2 dr d\theta ; dT = r dF = \frac{\mu \Delta\omega}{a} r^3 dr d\theta$$

Integrating

$$T = \int_0^{2\pi} \int_0^R dT = \frac{\mu \Delta\omega}{a} \int_0^{2\pi} \int_0^R r^3 dr d\theta = \frac{2\pi \mu \Delta\omega}{a} \int_0^R r^3 dr = \frac{\pi \mu \Delta\omega R^4}{2a}$$

$$P_o = T \omega_o = \frac{\pi \mu \omega_o \Delta\omega R^4}{2a} \quad (\text{power transmitted})$$

$$s = \frac{\Delta\omega}{\omega_i} = \frac{2aT}{\pi \mu R^4 \omega_i}$$

Efficiency is $\eta = \frac{\text{Power out}}{\text{Power in}} = \frac{T \omega_o}{T \omega_i} = \frac{\omega_o}{\omega_i}$. But $\omega_o = \omega_i - \Delta\omega$, so

$$\eta = \frac{\omega_i - \Delta\omega}{\omega_i} = 1 - \frac{\Delta\omega}{\omega_i} = 1 - s$$